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Numerical Visualization of Free Surface Oscillation Predicted with Arbitrary Lagrangian-Eulerian Method

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***Abstract*:** A numerical prediction method has been proposed to predict non-linear free surface oscillation in a three-dimensional container. The fluid motions are numerically predicted with Navier-Stokes equations discretized in a Lagrangian scheme with sufficient numerical accuracy. The profile of a free surface is precisely represented with three-dimensional body-fitted coordinates (BFC), which are regenerated in each computational step on the basis of the arbitrary Lagrangian-Eulerian (ALE) formulation. The computational method was applied to non-linear sloshings and transitions from sloshing to swirling motions. The predicted free surface motions were visualized as sequential image files and animations to understand their dynamic futures.

***Keywords*:** non-linear sloshing, moving boundary, body-fitted coordinates, arbitrary Lagrangian-Eulerian, numerical visualization.

1. Introduction

The dynamic oscillation of liquid with a free surface has long been of interest in a variety of engineering fields. In particular, non-linear sloshings with large amplitudes and more complicated swirling motions are sometimes considered as important phenomena associated with engineering design and assessment.

In the present study, a computational technique has been proposed to predict non-linear sloshing problems in an arbitrarily-shaped three-dimensional container. The liquid motions are described with Navier-Stokes equations instead of velocity potential models widely adopted in usual computational methods. The profile of a liquid surface is accurately represented by the three-dimensional curvilinear coordinates which are regenerated in each computational step on the basis of the arbitrary Lagrangian-Eulerian (ALE) formulation. Since the boundary conditions near the free surface can be implemented completely in the computational space, the present method is particularly advantageous to the usual techniques in which Eulerian computational grids are adopted. Moreover, in this transformed space, the governing equations are discretized on a Lagrangian scheme in which numerical accuracy is preserved at a sufficient level.

In order to demonstrate the applicability of the present computational technique, numerical simulations were performed for two-dimensional benchmark problems and three-dimensional sloshings within cylindrical tanks in various conditions. Some of the predicted results were numerically visualized as sequential image files and animations, which allow us to understand the dynamic features of free surface oscillations.

2. Numerical Procedure

2.1 Grid Generation

The non-orthogonal curvilinear coordinates are regenerated in each computational time step in order to represent adequately the shape of the free surface at every moment, which is deformed unsteadily and non-uniformly. In contrast to the Lagrangian grid generation, the ALE formulation allows us to create curvilinear coordinates independently of the liquid motion. Thus, the velocity of the computational grid point may not coincide with that of the liquid. Therefore, once a shape of the free surface is specified, the corresponding curvilinear coordinates are generated in an arbitrary-shaped three-dimensional liquid region taking this profile as one of the boundary conditions. The governing equations and the numerical method to generate BFC are the same as those proposed by Ushijima (1994).

2.2 Governing Equations and Discretization

The incompressible liquid motion with a free surface is described by Navier-Stokes equations instead of velocity potential models, so that the three-dimensional flows can be treated more generally as rotational and viscid liquid behaviors. The governing equations which are transformed into the computational space are given by

$$\frac{Du_i}{Dt} = -\frac{1}{\rho}\frac{\partial p}{\partial \xi_m}\frac{\partial \xi_m}{\partial x_i} + F_i + \quad \left(\frac{\partial^2 u_i}{\partial \xi_m \partial \xi_n}\frac{\partial \xi_m}{\partial x_j}\frac{\partial \xi_n}{\partial x_j} + P_m \frac{\partial u_i}{\partial \xi_m} \right) \tag{1}$$

Here u_i, F_i, p, ρ, and   are velocity and external force in x_i direction, pressure, fluid density and kinematic viscosity, respectively. The coordinates ξ_m are defined in the transformed space and P_m is a control function governing the mesh intervals in the physical space. The gravity and the forced acceleration imposed to cause sloshing motions are included in F_i in Eq. (1). Since the ALE formulation is employed, the Lagrangian differential operator in Eq. (1) is given by the following form (Ushijima, 1996):

$$\frac{D}{Dt} = \frac{\partial}{\partial \tau} + \left(U_m - U_{0m} \right) \frac{\partial}{\partial \xi_m} \tag{2}$$

where t and τ are times in the physical and computational spaces, respectively, which are set to be identical. The contravariant velocity components U_m and U_{0m} represent the velocity of the liquid and that of the computational grid point, respectively.

The transformed momentum equations are discretized on a Lagrangian scheme in the computational space. For convenience, Eq. (1) may be expressed in the following form:

$$\frac{Du_i}{Dt} = -PG_i + F_i + D_i \tag{3}$$

where PG_i and D_i stand for the pressure gradient and diffusion terms in Eq. (1), respectively. Equation (3) can be discretized in the following form as proposed by Ushijima (1994):

$$u_i^{n+1} = u'^{n}_i + \left[-PG_i^{n+1} + \left(\frac{3}{2} D'^{n}_i - \frac{1}{2} D''^{n-1}_i \right) \right] \Delta t \tag{4}$$

Here the superscript n means the computational time-step number, and prime and double prime stand for the spatial location at the upstream points at n and n 1 steps, respectively.

The first term on the right hand side of Eq. (4), corresponding to the convection term, is calculated with a local cubic spline interpolation (LCSI) method proposed by Ushijima (1994). It has been proved that the LCSI allows us to have more accurate results than a third-order upwind difference method in pure advection problems (Ushijima, 1994).

The kinematic free surface condition is determined from the fact that the surface moves with the liquid in the physical space as

$$\frac{\partial h}{\partial t} + u_{si}\frac{\partial h}{\partial x_i} = u_{s3} \quad [i = 1, 2] \tag{5}$$

where h is the free surface height and subscript s means that the corresponding values are defined at the grid point on the free surface.

When the viscous stresses on a liquid-gas interface are negligible, the stress conditions on free surfaces are determined by the following two equations:

$$\tau_i \sigma_{ij} n_j = 0 \tag{6}$$

$$n_i \sigma_{ij} n_j = Sk \tag{7}$$

where S and k mean the coefficient of surface tension and the curvature of the free surface respectively. The unit vectors n_i and τ_i are normal and tangential to the free surface and the stress tensor σ_{ij} is defined by

$$\sigma_{ij} = -p_0 \delta_{ij} + \mu \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \tag{8}$$

where p_0 is the atmospheric pressure and μ is the dynamic viscosity. The detailed forms of these boundary conditions in the transformed space are indicated by Ushijima (1998). In the present paper, it is assumed that the atmospheric pressure equals zero and that the effect of surface tension can be neglected.

2.3 Solution Procedure

The main solution procedures are summarized as follows. Firstly, initial free surface profile and other necessary initial conditions are specified. Then the three-dimensional curvilinear coordinates, which are coincident with the provided free surface and the other fixed boundary shapes, are generated. In the computational space corresponding to the generated coordinates, numerical procedure for liquid calculations is performed; the convection and diffusion terms are firstly evaluated on a Lagrangian scheme and approximate velocity is derived with these values, assuming the pressure field to be given by a hydrostatic pressure distribution. After the converged correction pressure is obtained from the iterative calculations, correction velocity components and free surface levels are finally derived at a new computational step. When the unsteady numerical prediction still proceeds, the free surface profile is updated and new curvilinear coordinates are generated again for it. In this way, unsteady numerical procedure continues until the appointed time.

2.4 Numerical Visualization of Free Surface

The development of a numerical visualization technique is also important in the present study, since the predicted results are three-dimensional and they are obtained in unsteady conditions.

In the computation of sloshings, free surface profiles are represented by curvilinear coordinates regenerated in each computational step. Thus, the coordinates are saved on a hard disk at the appointed time steps during the computation. The saved data are visualized after the computation with a post processing program, which utilizes OpenGL libraries (Davis and Woo, 1993). With this program, calculated results are rendered on a computer display with perspective projection and it allows us to translate and rotate the objects and also change their scales through mouse operations.

While the profile of a free surface may be drawn as a smoothly shaded model, texture mapping is much effective to make the graphics more realistic. Thus, the free surface is displayed using environment mapping, which renders an object as it were perfectly reflective; the colors on the surface are those reflected to the eye from its surroundings (Davis and Woo, 1993). Some of the predicted results will be shown later with this environment mapping.

3. Application of Prediction Method

3.1 Free Oscillation

The numerical analysis of the free oscillation of a liquid with a small amplitude allows us to confirm that the numerical technique satisfies the necessary specifications, such as mass and momentum conservation. The present example has the same conditions as adopted by Ramaswamy (1990); the two dimensional rectangular container, as shown in Fig. 1, is 1.0 units in width and 1.0 units in height, and the gravity acts downward with a unit magnitude. The initial profile of the free surface is given by

$$h = 1.0 + a \sin\left[\pi\left(0.5 - x_1\right)\right] \tag{9}$$

where the amplitude of the antisymmetric natural mode equals 0.01 units.

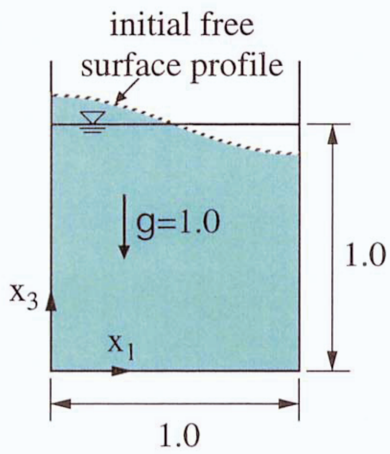


Fig. 1. Definition sketch.

Figure 2(a) shows the time history of the free surface displacements at both ends, $x_1 = 0$ and 1.0, calculated with the non-viscosity condition, in which free slip condition is given on all solid boundaries of the container and liquid viscosity is set to be zero. As shown in this figure, no numerical damping effects are found and the conservation of mass and momentum is completely satisfied. On the other hand, Figure 2(b) shows the time history of the amplitude calculated with the viscous condition, in which liquid velocity on the solid boundaries is zero and kinematic viscosity of the liquid equals 0.01 units. The adequate attenuation of the wave amplitudes is observed in the result, which has similar tendency to the results presented by Ramaswamy (1990).

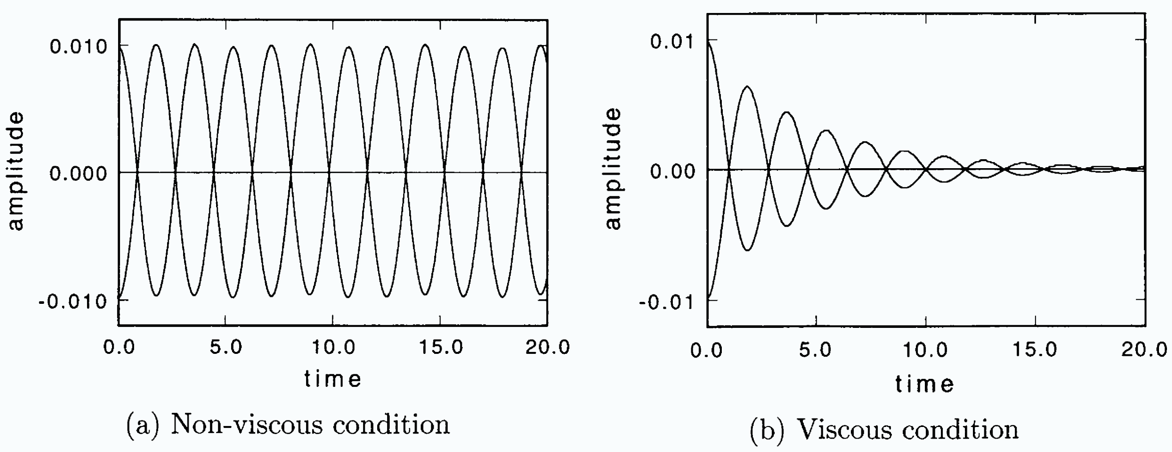


Fig. 2. Time history of free surface displacement.

3.2 Nonlinear Oscillation

The following example has been chosen in many previous numerical studies as done by Harlow and Welch (1966), Ramaswamy (1990), Takizawa et al. (1992) and others, in order to demonstrate the validity of the computing technique for highly nonlinear oscillation. As shown in Fig. 3, the rectangular container is 4.8 units in width and 4.0 units in height. A gravity acceleration of one unit acts downwards and the cosine pressure impulse is imposed on the free surface of the rest liquid in the container. The pressure pulse is defined by

$$p_0(t) = A\delta(t)\cos(kx_1) \tag{10}$$

where $\delta\ (t)$ is the dirac delta function and the amplitude A equals 1.0 units. The disturbance wave number k is given by $2\pi/9.6$. The liquid kinematic viscosity is defined by 0.01 units and free slip condition is imposed on the solid boundaries of the container.

The time history of the free surface displacements is shown in Fig. 4, in which the amplitudes of the linear analysis and the numerical results calculated by Harlow and Welch (1966) are also presented. The present results

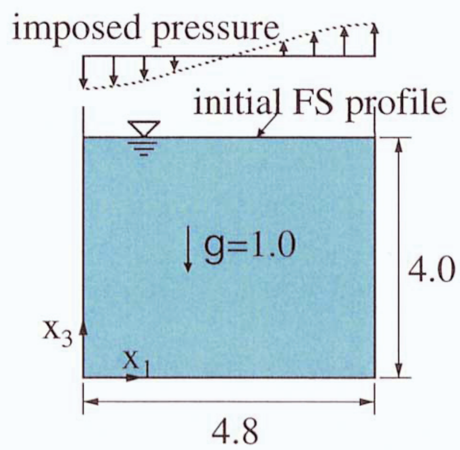


Fig. 3. Dimensions of a container.

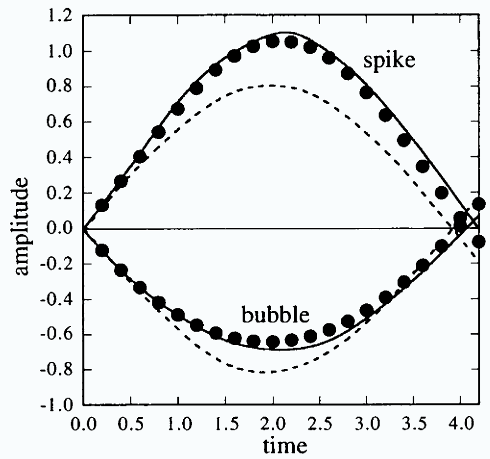


Fig. 4. Time history of free surface oscillation (present results, Harlow and Welch, --- linear analysis).

in Fig. 4 shows the highly nonlinear spike and bubble, which are quite similar to the results of Harlow and Welch (1966).

3.3 Forced Horizontal Oscillation

Figure 5 illustrates the definition sketch, in which Cartesian coordinates and dimensions are indicated, where R and H correspond to the radius of a cylindrical tank and liquid depth in static condition, respectively. In a cylindrical tank with $R = H = 0.5$ m, the liquid with the kinematic viscosity of 0.01 cm^2/s is subject to the forced acceleration in x_1 direction as given by

$$a_1(t) = -X_1\omega^2 \sin(\omega t) \tag{11}$$

where the amplitude X_1 and angular frequency ω in Eq. (11) are 2 mm and 10.22 radian/sec, respectively. This external vibration coincides with the (1, 2) mode of the natural frequency (Lamb, 1932). Figure 6 shows the sequential images of the predicted free surface oscillation. The free surface images in Fig. 6 are derived with the environment mapping described in Section 2.4, together with the bottom and side boundaries displayed as wire-frame surfaces.

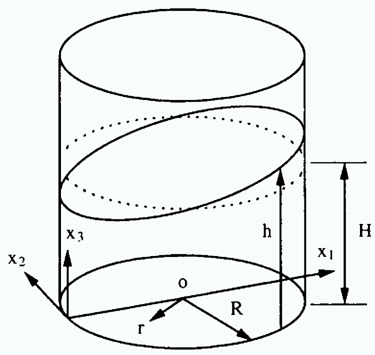


Fig. 5. Definition sketch.

3.4 Swirling Motion of Free Surface

It has been reported that when an axisymmetrical tank of liquid is subject to a harmonic vibration in a single horizontal direction, the free surface motion may rotate harmonically or non-harmonically around the vertical axis of the tank. This swirling motion of waves was observed by Hutton (1963) in a cylindrical tank laterally oscillated at the frequencies just below the lowest natural frequency. In the present investigation, transition from non-linear sloshing to swirling motions in a cylindrical tank is numerically predicted by setting up the same initial trigger in

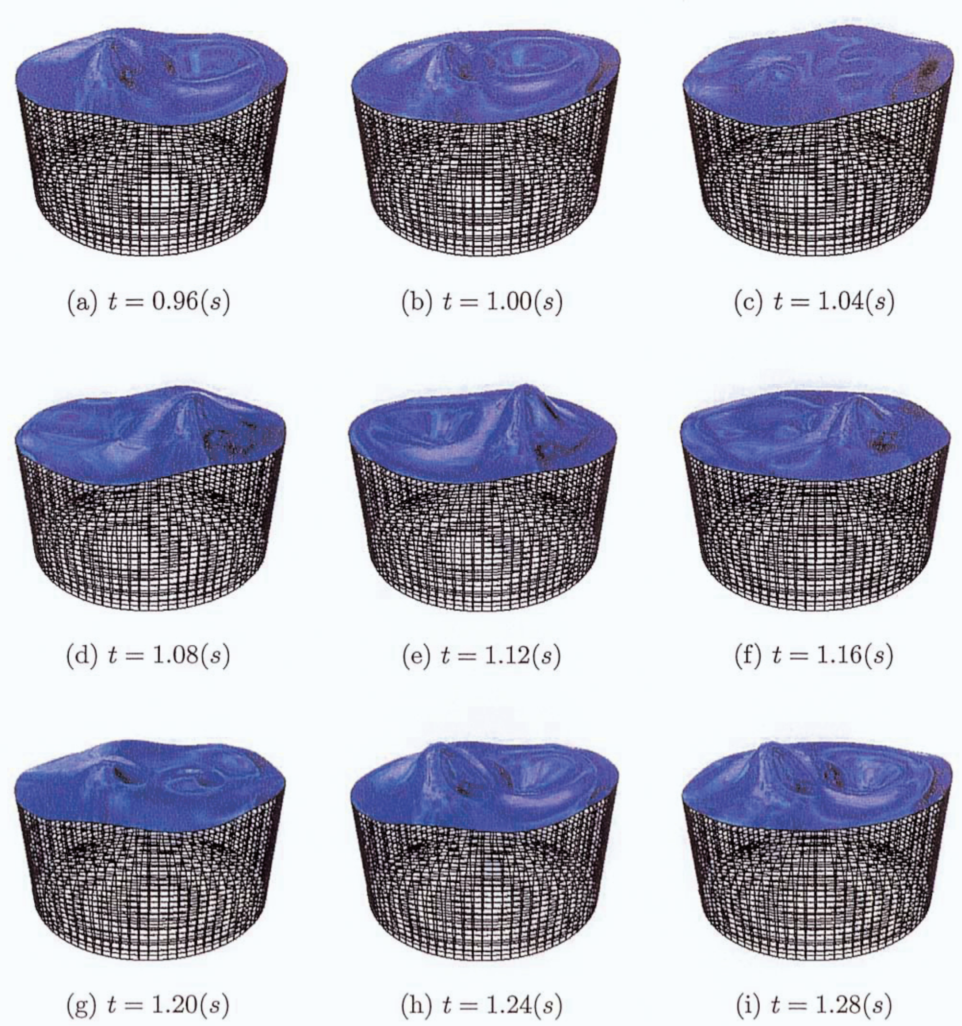


Fig. 6. Visualized (1, 2) mode sloshing.

x_2-direction, as employed by Tanaka and Nakayama (1991):

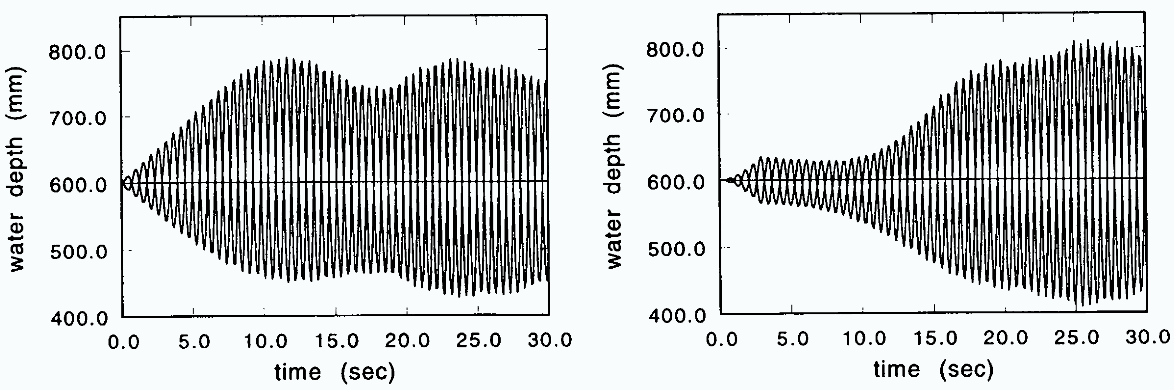
$$a_2(t) = \begin{cases} X_2 \sin(2\pi ft) & [0 \le t \le 3(s)] \\ 0 & [3(s) < t] \end{cases} \tag{12}$$

while the following harmonic vibration is continuously imposed in x_1-direction:

$$a_1(t) = X_1 \sin(2\pi ft) \quad [0 \le t] \tag{13}$$

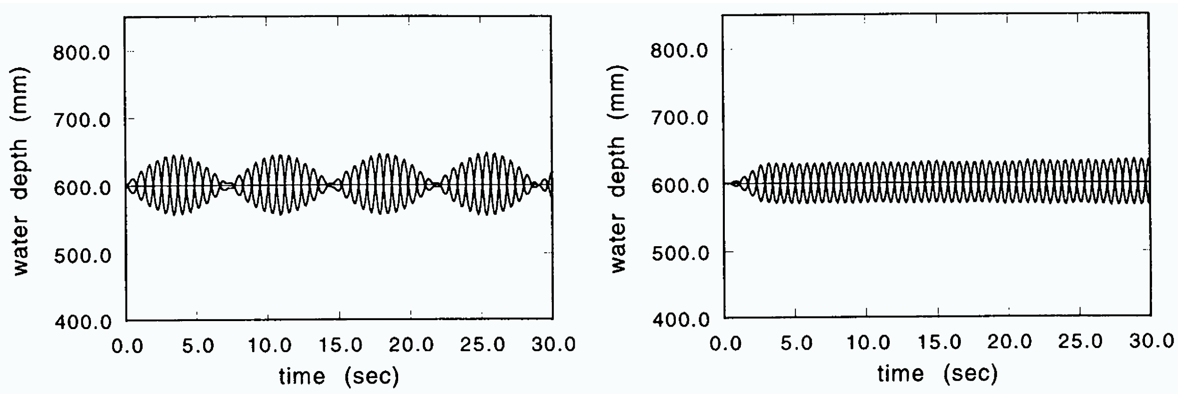
where $X_1 =\ \ 0.0178g$ and $X_2 = X_1 \sin(\pi t/6)$. The geometries of the cylindrical tank are $R = 0.5$ m and $H = 0.6$ m. While the natural frequency of (1,1) mode equals 0.944 Hz in the present geometries, the calculation is performed by setting $f = 0.940$ Hz, as done by Tanaka and Nakayama (1991).

Figures 7(a) and (b) show the surface displacements at $(x_1, x_2) = (0, 0)$ and $(2R, 0)$, and $(x_1, x_2) = (R,\ \ R)$ and (R, R), respectively. On the plane of principal excitation, $x_1\ \ x_3$ plane, a large non-linear response gradually develops, as shown in Fig. 7(a). On the other hand, while relatively small oscillation continues on $x_2\ \ x_3$ plane in the initial stage, as shown in Fig. 7(b), the amplitudes becomes larger at around t = 10s, which indicates the transition to swirling motion of the free surface. In contrast, when the frequency of principal force is set at f = 0.796 Hz, no transition to swirling motions appears as shown in Fig. 8.



(a) $(x_1, x_2) = (0, 0)$ and $(2R, 0)$ (b) $(x_1, x_2) = (R,\quad R)$ and (R, R)

Fig. 7. Time history of free surface displacement at f 0.940 Hz.



(a) $(x_1, x_2) = (0, 0)$ and $(2R, 0)$ (b) $(x_1, x_2) = (R,\quad R)$ and (R, R)

Fig. 8. Time history of free surface displacement at f 0.796 Hz.

4. Concluding Remarks

A numerical prediction method has been proposed to predict non-linear free surface oscillation in a three-dimensional container. The fluid motions are numerically predicted with Navier-Stokes equations and the profile of a free surface is precisely represented with 3D BFC on the basis of an ALE method. The computational method was applied to two-dimensional benchmark computations and non-linear sloshing and swirling motions in three-dimensional cylindrical tanks. As a result, a reasonable applicability has been confirmed in the present computational technique. The predicted free surface motions were numerically visualized with environment mapping, which allows us to capture their dynamic features.

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